复习格林函数：

因果格林函数：

$$G(x,t;x',t') = -\frac{i}{\hbar}\langle T\{\tilde{\Psi}_\alpha(x,t)\,\tilde{\Psi}_\beta(x',t')\}\rangle$$

$\boxed{\text{Heisenberg picture}}$

推迟格林函数：

$$G^R(x,t;x',t') = -\frac{i}{\hbar}\Theta(t-t')\langle\langle[\tilde{\Psi}(x,t),\tilde{\Psi}^+(x',t')]_\eta\rangle\rangle$$

超前格林函数：

(费米子，反对易
玻色子，对易)

$$G^A(x,t;x',t') = \frac{i}{\hbar}\Theta(t'-t)\langle\langle[\tilde{\Psi}(x,t),\tilde{\Psi}^+(x',t')]_\eta\rangle\rangle$$

$G^>$： $G^>(\vec{k},t-t') = -i\langle\langle\tilde{a}_{\vec{k}}(t)\,\tilde{a}^+_{\vec{k}}(t')\rangle\rangle$

$G^<$： $G^<(\vec{k},t-t') = \pm i\langle\langle\tilde{a}^+_{\vec{k}}(t')\,\tilde{a}_{\vec{k}}(t)\rangle\rangle$ (费米子/玻色子)

谱函数： $A(\vec{k},\omega)$

谱函数表示：

定义： $-iA(\vec{k},\omega) = G^>(\vec{k},\omega) - G^<(\vec{k},\omega)$

由 $G^>(\vec{k},\omega) = \mp G^<(\vec{k},\omega)e^{\beta\omega}$

$$\Longrightarrow \begin{cases} G^>(\vec{k},\omega) = -i(1\mp n(\omega))A(\vec{k},\omega) \\ G^<(\vec{k},\omega) = \pm i\,n(\omega)A(\vec{k},\omega) \end{cases}$$

$$\boxed{n(\omega) = \frac{1}{e^{\beta\omega}\mp 1}}$$

因果格林函数与 $G^{\gtrless}$ 的关系：

$$G(\vec{k}, t-t') = \theta(t-t')\,G^{>}(\vec{k}, t-t') + \theta(t'-t)\,G^{<}(\vec{k}, t-t')$$

$$G(\vec{k},\omega) = \int e^{i\omega t'}\left(\theta(t')\,G^{>}(\vec{k},t') + \theta(-t')\,G^{<}(\vec{k},t')\right)dt'$$

$$= \frac{1}{2\pi i}\int d\omega' \int dt' \left(\frac{G^{>}(\vec{k},t')}{\omega' - i0_+} - \frac{G^{<}(\vec{k},t')}{\omega' + i0_+}\right)e^{i(\omega+\omega')t'}$$

$$= \frac{1}{2\pi i}\int d\omega' \left(\frac{G^{>}(\vec{k},\omega')}{\omega'-\omega-i0_+} - \frac{G^{<}(\vec{k},\omega')}{\omega'-\omega+i0_+}\right)$$

$$\Rightarrow \quad G(\vec{k},\omega) = \int \frac{d\omega'}{2\pi} A(\vec{k},\omega')\left(\frac{1\mp n(\omega')}{\omega-\omega'+i0_+} \pm \frac{n(\omega')}{\omega-\omega'-i0_+}\right)$$

元激发能谱与寿命

费米子寿命（跃迁到其它态上）

玻色子寿命（湮灭）

自由粒子：$G^{>}(\vec{k},t) = -i\langle\langle \tilde{a}_{\vec{k}}(t)\,\tilde{a}^{\dagger}_{\vec{k}}(0)\rangle\rangle = -i e^{-i\varepsilon_{\vec{k}} t}\underbrace{\langle\langle \tilde{a}_{\vec{k}}(0)\,\tilde{a}^{\dagger}_{\vec{k}}(0)\rangle\rangle}_{1-n_{\vec{k}}}$

费米子

频率空间：$$G^{>}(\vec{k},\omega) = \int dt\, e^{i\omega t} G^{>}(\vec{k},t)$$

$$= -i\int dt\, e^{i(\omega-\varepsilon_{\vec{k}})t}(1-n_{\vec{k}})$$

$$= -2\pi i\,\delta(\omega-\varepsilon_{\vec{k}})\,(1-n_{\vec{k}})$$

$$G^{<}(\vec{k},\omega) = i\int dt\, e^{i(\omega-\varepsilon_{\vec{k}})t}\, n_{\vec{k}}$$

$$= 2\pi i\,\delta(\omega-\varepsilon_{\vec{k}})\, n_{\vec{k}}$$

谱函数：（响应函数）

$$A(\vec{k},\omega) = \underline{2\pi\,\delta(\omega-\varepsilon_{\vec{k}})} = i\left[G^{<}(\vec{k},\omega) - G^{>}(\vec{k},\omega)\right]$$

用谱函数求因果格林函数：

$$G(\vec{k},\omega) = \int \frac{d\omega'}{2\pi} A(\vec{k},\omega')\left[\frac{1\mp n(\omega')}{\omega-\omega'+i0_+} \pm \frac{n(\omega')}{\omega-\omega'-i0_+}\right]$$

（费米子/玻色子）$\boxed{\text{费米子符号在上}}$

$$G(\vec{k},\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} 2\pi\,\delta(\omega'-\varepsilon_{\vec{k}})\left[\frac{1-n(\omega')}{\omega-\omega'+i0_+} + \frac{n(\omega')}{\omega-\omega'-i0_+}\right]$$

$$= \frac{1-n(\varepsilon_k)}{\omega-\varepsilon_{\vec{k}}+i0_+} + \frac{n(\varepsilon_k)}{\omega-\varepsilon_{\vec{k}}-i0_+}$$

（无穷寿命）

零温下： $n(\varepsilon_k) = \Theta(\varepsilon_F - \varepsilon_k)$

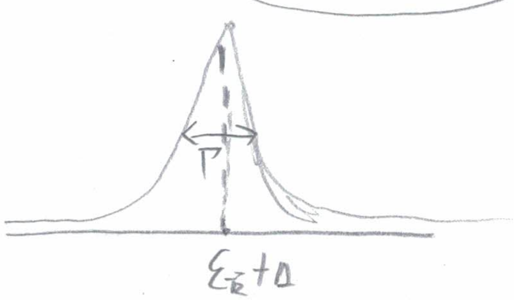
$$G(\vec{k},\omega) = \frac{\Theta(\varepsilon_k-\varepsilon_F)}{\omega-\varepsilon_{\vec{k}}+i0_+} + \frac{\Theta(\varepsilon_F-\varepsilon_k)}{\omega-\varepsilon_{\vec{k}}-i0_+}$$

作业：$\boxed{G^R(\vec{k},\omega) \text{ 和 } G^A(\vec{k},\omega)}$

元激发的寿命：（假定一个谱函数）

$$A(\vec{k},\omega) = \frac{\Gamma}{(\omega-\varepsilon_{\vec{k}}-\Delta)^2+\frac{\Gamma^2}{4}}$$

Lorentz 型



展宽为Γ

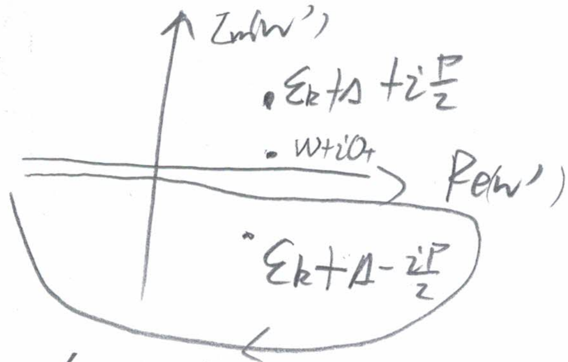
$$G(\vec{k},\omega)=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Gamma}{(\omega'-\epsilon_{\vec{k}}-\Delta)^2+\frac{\Gamma^2}{4}}\left[\frac{\theta(\omega'-\epsilon_F)}{\omega-\omega'+i0_+}+\frac{\theta(\epsilon_F-\omega')}{\omega-\omega'-i0_+}\right]$$

$$=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Gamma}{(\omega'-\epsilon_{\vec{k}}-\Delta)^2+\frac{\Gamma^2}{4}}\frac{1}{\omega-\omega'+i0_+\,\mathrm{sgn}(\omega)}$$

奇点：

$$\omega'=\epsilon_{\vec{k}}+\Delta\pm\frac{i\Gamma}{2}$$

$$\omega'=\omega+i0_+\,\mathrm{sgn}(\omega)$$



作业，选上围道

下围道：
$$\frac{1}{i}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\left[\frac{1}{\omega'-(\epsilon_{\vec{k}}+\Delta)-\frac{i\Gamma}{2}}-\frac{1}{\omega'-(\epsilon_{\vec{k}}+\Delta)+\frac{i\Gamma}{2}}\right]\frac{1}{\omega-\omega'+i0_+\,\mathrm{sgn}(\omega)}$$

$$=\frac{1}{\omega-(\epsilon_{\vec{k}}+\Delta-\frac{i\Gamma}{2})+i0_+}$$

$$\Rightarrow G(\vec{k},\omega)=\frac{1}{\omega-(\epsilon_{\vec{k}}+\Delta-\frac{i\Gamma}{2})}$$

涨落耗散定理：

先求 $G^>$, $G^<$

$$G^{>}(\vec{k},t) = -i\langle\langle a_{\vec{k}}(t)\, a^{+}_{\vec{k}}(0)\rangle\rangle$$

零时刻产生元激发 $\longrightarrow$ t时刻湮灭的几率幅

$$G^{>}(\vec{k},t) = -i\int\frac{d\omega}{2\pi}\, e^{-i\omega t}\,\frac{\Gamma}{(\omega-\varepsilon_k-\Delta)^2+\frac{\Gamma^2}{4}}\,(1-n_{\vec{k}})$$

$$= -i\, e^{-i(\varepsilon_{\vec{k}}+\Delta-i\frac{\Gamma}{2})t}\,(1-n_{\vec{k}})$$

kubo 线性响应理论 $H = H_0 + H'_I$

$$i\hbar \frac{\partial \hat{\rho}^I}{\partial t} = [\hat{H}'_I, \hat{\rho}_I]$$

$$\hat{\rho}(t) = \rho_I(-\infty) + \frac{1}{i\hbar}\int_{-\infty}^{t} dt' [H'_I(t'), \rho(-\infty)]$$

$$+ \left(\frac{1}{i\hbar}\right)^2 \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' [H'_I(t'), [H'_I(t''), \rho_{(-\infty)}]] + \cdots$$

物理量：$F = Tr(\hat{\rho}(t)\hat{F})$

线性响应：

$$F = \langle\langle F \rangle\rangle_0 + \frac{1}{i\hbar}\int_{-\infty}^{t} \langle\langle [F_I(t), H'_I(t')] \rangle\rangle_0 dt'$$

① 磁响应

② 电响应

③ 温度响应

① 电子的磁响应

$H = H_0 + H'$

$H' = -\int d\vec{r}\, \underline{\vec{m}(\vec{r})} \cdot \vec{h}(\vec{r}, t)$

磁场 → 电子变化 → 磁矩变化

$$M_i(\vec{r}, t) = \langle\langle M_i(\vec{r}, t) \rangle\rangle_0 + \frac{1}{i\hbar}\int_{-\infty}^{t} dt' \langle\langle [M_i(\vec{r}, t), -\int d\vec{r}' m_j(\vec{r}', t') \cdot h_j(\vec{r}', t')] \rangle\rangle_0$$

$$M_i(\vec{r},t)=\frac{1}{i\hbar}\int_{-\infty}^{t}dt'\langle\langle[M_i(\vec{r},t),-\int d\vec{r}'m_j(\vec{r}',t')H_j(\vec{r}',t')]\rangle\rangle_0$$

$$=\frac{i}{\hbar}\int_{-\infty}^{+\infty}dt'\int d\vec{r}'\underbrace{\Theta(t-t')\langle\langle[m_i(\vec{r},t),m_j(\vec{r}',t')]\rangle\rangle_0}_{\text{推迟格林函数}}H_j(\vec{r}',t')$$

定义磁响应率： $\chi_{ij}(\vec{r}-\vec{r}',t-t')=i\Theta(t-t')$

$$\times\langle\langle[m_i(\vec{r},t),m_j(\vec{r}',t')]\rangle\rangle_0$$

$$M_i(\vec{r},t)=\int_{-\infty}^{+\infty}dt'\int d\vec{r}'\chi_{ij}(\vec{r}-\vec{r}',t-t')H_j(\vec{r}',t')$$

求线性响应，实际是求推迟格林函数（响应率）

$$\chi_{ij}(\vec{q},\omega)=\int d\vec{r}dt\,e^{-i\vec{q}\cdot\vec{r}+i\omega t}\chi_{ij}(\vec{r},t)$$

$$=i\int_0^{\infty}dt\,\langle\langle[m_i(\vec{q},t),m_j(-\vec{q},0)]\rangle\rangle_0e^{i\omega t}$$

$$=i(\gamma\hbar)^2\int_0^{+\infty}dt\,\langle\langle[\hat{S}_i(\vec{q},t),\hat{S}_j(-\vec{q},0)]\rangle\rangle_0e^{i\omega t}$$

$\hat{S}(\vec{q},t)\overset{?}{=}$ $\qquad\hat{S}(\vec{r})=\frac{1}{2}\sum_i\hat{\sigma}_i\delta(\vec{r}-\vec{r}_i)$

$$\hat{S}(\vec{q}) = \int \frac{d\vec{r}}{(2\pi)^{\frac{3}{2}}} \, S(\vec{r}) \, e^{-i\vec{q}\cdot\vec{r}}$$

$$= \frac{1}{2} \sum_i \vec{\sigma}_i \, e^{-i\vec{q}\cdot\vec{r}} = \frac{1}{2} \int d\vec{r} \; \hat{\Psi}^{+}(\vec{r}) \, \vec{\sigma} \, \hat{\Psi}(\vec{r}) \, e^{-i\vec{q}\cdot\vec{r}}$$

$$= \sum_{\vec{k},\alpha\beta} \frac{1}{2} \sigma_{\alpha\beta} \, \hat{a}^{+}_{\vec{k}+\vec{q},\alpha} \, \hat{a}_{\vec{k},\beta}$$

作业1：求零温 自由粒子的超前，推迟格林函数

解：$G^R(x,t;x',t') = -\frac{i}{\hbar}\theta(t-t')\langle\langle[\Psi(x,t)\,\Psi^+(x',t')]_\eta\rangle\rangle$

$\Psi(x,t) = \sum_{\vec{k}} \hat{a}_{\vec{k}}\, e^{i(\vec{k}\cdot\vec{r}-\varepsilon_{\vec{k}} t)}$

$\sum_{\vec{k}\vec{k}'}\langle\langle\{\hat{a}_{\vec{k}},\hat{a}^+_{\vec{k}'}\}\rangle\rangle e^{i(\vec{k}\cdot\vec{r}-\vec{k}'\cdot\vec{r}')}\, e^{i(\varepsilon_{\vec{k}'} t'-\varepsilon_{\vec{k}} t)/\hbar}$

$= \sum_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}\, e^{-i\varepsilon_{\vec{k}}(t-t')/\hbar}$

则 $G^R(\vec{k},t) = \left(-\frac{i}{\hbar}\right)\theta(t)\, e^{-i\varepsilon_{\vec{k}} t/\hbar}$

$\because \theta(t) = \int \frac{d\omega'}{2\pi i}\frac{e^{i\omega' t}}{\omega'-i0_+}$,

$\therefore G^R(\vec{k},\omega) = -\frac{i}{\hbar}\int\frac{d\omega'}{2\pi i}\int dt'\,\frac{e^{i(\omega'-\varepsilon_{\vec{k}}/\hbar+\omega)t'}}{\omega'-i0_+}$

$= -\frac{i}{\hbar}\cdot\frac{1}{i}\quad\frac{1}{\varepsilon_{\vec{k}}/\hbar-\omega-i0_+}$

$G^R(\vec{k},\omega) = \frac{1}{\omega-\varepsilon_{\vec{k}}+i0_+}$

由 $\theta(-t) = \int -\frac{d\omega'}{2\pi i}\frac{e^{i\omega' t}}{\omega'+i0_+}$

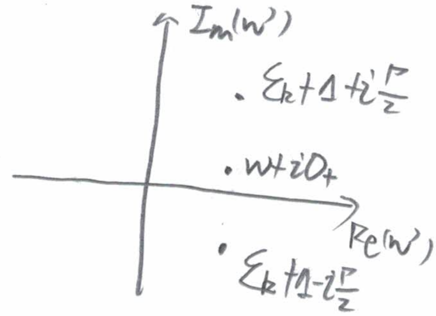
$\therefore G^A(\vec{k},\omega) = \frac{i}{\hbar}\int\frac{d\omega'}{-2\pi i}\int dt'\,\frac{e^{i(\omega'-\varepsilon_{\vec{k}}/\hbar+\omega)t'}}{\omega'+i0_+}$

$= \frac{1}{\omega-\varepsilon_{\vec{k}}-i0_+}$

作业2：

因果格林函数：$G(\vec{k},\omega)=\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Gamma}{(\omega'-\varepsilon_k-\Delta)^2+\frac{\Gamma^2}{4}}\frac{1}{\omega-\omega'+i0_+\mathrm{sgn}(\omega')}$

选上围道计算



解：$G(\vec{k},\omega)= i\mathrm{Res}(\varepsilon_k+\Delta+i\frac{\Gamma}{2})+i\mathrm{Res}(\omega+i0_+)$

$$= i\cdot\frac{1}{\omega-(\varepsilon_k+\Delta+i\frac{\Gamma}{2})}\cdot\frac{1}{i}$$

$$+ i\cdot\frac{-\Gamma}{(\omega-\varepsilon_k-\Delta)^2+\frac{\Gamma^2}{4}}$$

$$= -\cancel{\frac{1}{\omega-(\varepsilon_k+\Delta)^2-\frac{i\Gamma}{2}}}+\frac{1}{\omega-(\varepsilon_k+\Delta)+\frac{i\Gamma}{2}}+\cancel{\frac{1}{\omega-(\varepsilon_k+\Delta+i\frac{\Gamma}{2})}}$$

$$= \frac{1}{\omega-(\varepsilon_k+\Delta)+\frac{i\Gamma}{2}}$$